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LAGRANGE INTERPOLATION AND BOUNDARY-VALUE PROBLEMS

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ABSTRACT. It is known that some boundary-value problems give rise to Lagrange-type interpolation series that can be used to reconstruct entire functions from their samples at the eigenvalues of any such problem. Such boundary-value problems, of which regular Sturm Liouville boundary-value problems are prototype, are said to have the Lagrange-type interpolation property.

It was conjectured that any regular, self-adjoint, eigenvalue problem associated with nth order linear differential operator with simple eigenvalues has the Lagrange-type interpolation property. In 1994, P.L. Butzer and G. Schöttler proved that conjecture, but a year later Annaby pointed out an error in their paper and gave an alternative proof which is not only imprecise, but also deals with a very special case in which the problem is assumed to be one dimensional.

The aim of this article is to fill the gaps in these papers by providing an alternative proof.

1. Introduction.

1.1. *Sampling theorems and Lagrange interpolation.* The Whittaker-Shannon-Kotel'nikov (WSK) sampling theorem plays an important role not only in harmonic analysis and approximation theory, but also in communication engineering. The theorem can be stated as follows:

Theorem 1.1 (Whittaker-Shannon-Kotel'nikov). *If a function f is band-limited to $[-\sigma, \sigma]$, i.e., it is representable as*

$$f(t) = \int_{-\sigma}^{\sigma} e^{-ixt} g(x)\, dx \quad t \in \mathbf{R}, \tag{1.1}$$

Key words and phrases. Self-adjoint boundary-value problems, Lagrange interpolation, Shannon sampling theorem, Kramer sampling theorem.
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