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VOLTERRA INTEGRO-DIFFERENTIAL EQUATIONS WITH ACCRETIVE OPERATORS AND NON-AUTONOMOUS PERTURBATIONS

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ABSTRACT. This paper is devoted to study a class of nonlinear scalar Volterra equations in general Banach spaces, with an m-accretive leading operator and a nonautonomous perturbation. We shall consider both the case of Lipschitz perturbations and the case of dissipative perturbations. We prove the existence of a generalized solution and discuss some useful estimates for it.

1. Introduction. The type of Volterra equations studied in this paper is the nonlinear evolution equation

(1.1)

$$\begin{cases} \dfrac{d}{dt}\left(k_0(u(t)-x)+\displaystyle\int_0^t k_1(t-s)(u(s)-x)\,ds\right)+G(u(t))=F(t,u(t)), \\ t\in(0,\infty), \quad u(0+)=x, \end{cases}$$

in a real Banach space X. Here, $k_0 \geq 0$ is a constant and k_1 is a real, nonnegative function that satisfy Hypothesis 1a) below, G is an accretive operator in X, see Hypothesis 1b), and we shall consider the operator $F(t,u)$ as a nonlinear, nonautonomous perturbation of the operator G, see Hypothesis 1c) for details.

Since the early 1970s, the case where $F(t,u)=f(t)$ has been under consideration; this problem has an interest also in our setting, and it shall be further discussed in Section 2.1. The next step in the literature was to consider functional perturbations of such a problem, compare [**4, 8**].

In this paper, on the contrary, we consider perturbation operators acting on X, but we can allow such operators to be nonautonomous. The study of (1.1) with the operator $F(t,u)$ is based on the results for

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