JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 18, Number 3, Fall 2006

PROLATE SPHEROIDAL WAVELETS IN HIGHER DIMENSIONS

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ABSTRACT. Prolate spheroidal wavelets (PS wavelets) based on the first prolate spheroidal wave function (PSWF), were recently introduced. They were shown to have many desirable properties lacking in other wavelets. In particular, the subspaces belonging to the associated MRA were shown to be closed under differentiation and translation. In this paper, we introduce prolate spheroidal wavelets in higher dimensions. They are similar to the one-dimensional versions in that they are based on an eigenfunction of an integral operator. But there is not, in general, an associated differential operator which is helpful in the one dimensional case for construction. Hence another method of construction must be used.

1. Introduction. The prolate spheroidal wave functions (PSWFs), $\{\varphi_{n,\sigma,\tau}(t)\}_{n\in Z}$, constitute an orthonormal basis of the space of σ-bandlimited functions on the real line. They are the eigenfunctions of an integral operator with the sinc function, $S(t) = \sin \pi t/\pi t$, as its kernel:

$$\int_{-\tau}^{\tau} \varphi_{n,\sigma,\tau}(x)\, \frac{1}{T}\, S\left(\frac{t-x}{T}\right) dx = \lambda_{n,\sigma,\tau}\, \varphi_{n,\sigma,\tau}(t), \tag{1}$$

where $T = \pi/\sigma$. They were obtained, in a series of papers by Slepian, Pollak and Landau at Bell Labs, as the solutions of an energy concentration problem which led to this integral equation. The problem was to find the normalized σ-bandlimited function with the maximum energy concentration on the interval $[-\tau, \tau]$. The solution is the first PSWF $\varphi_{0,\sigma,\tau}$; the function orthogonal to $\varphi_{0,\sigma,\tau}$ which possesses the maximum energy concentration on $[-\tau, \tau]$ is $\varphi_{1,\sigma,\tau}$, etc. They are also solutions to a Sturm-Liouville problem arising from the Helmholz equation on the prolate spheroid. Ergo the name.

Key words and phrases. Prolate spheroidal wave functions, wavelets, PS wavelets, bandlimited signal, Paley-Wiener space.
Received by the editors on May 3, 2005, and in revised form on December 27, 2005.