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VARIABLE COEFFICIENT TRANSMISSION PROBLEMS AND SINGULAR INTEGRAL OPERATORS ON NON-SMOOTH MANIFOLDS

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Dedicated to Professor K. Atkinson in recognition of his many contributions to the field of integral equations

1. Introduction. In this paper we discuss a new approach and an extension of the results in [**11**] regarding transmission boundary value problems and spectral theory for singular integral operators on Lipschitz domains. The main novelty here is the consideration of *variable coefficient* operators and systems which, in turn, requires a change in the strategy employed in [**11**]. In that paper, an approach based on the Serrin-Weinberger asymptotic theory, akin to the influential work of Dahlberg and Kenig [**9**], has been used. By further building on the work in [**11, 20, 34, 44**], here we develop an alternative approach, based on the regularity of the Neumann function, which is capable of handling variable coefficient operators of Schrödinger type on Lipschitz subdomains of Riemannian manifolds. One key feature of this approach is that it avoids the discussion of the asymptotic behavior at infinity for solutions of elliptic PDE's with bounded, measurable coefficients. In order to be more specific we shall now introduce some notation, starting with the geometric setting we have in mind.

Assume that $\mathcal{M}$ is a compact Riemannian manifold, of real dimension $n := \dim \mathcal{M} \geq 2$, equipped with a Lipschitz metric tensor $\mathfrak{g} := \sum g_{jk} dx_j \otimes dx_k$. Throughout the paper we let $d\mathcal{V} := g^{1/2} dx_1 \dots dx_n$, where $g := \det g_{jk}$, be the volume element on $\mathcal{M}$, and denote by

$$\Delta u := g^{-1/2} \sum_{j,k} \partial_j \big(g^{jk} g^{1/2} \, \partial_k u \big), \qquad (g^{jk})_{jk} := (g_{jk})^{-1}_{jk}, \tag{1.1}$$

the Laplace-Beltrami operator on $\mathcal{M}$.

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