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INTEGRAL OPERATORS AND DELAY DIFFERENTIAL EQUATIONS

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This paper is presented in honor of Kendall Atkinson.

ABSTRACT. The monodromy operator of a linear delay differential equation with periodic coefficients is formulated as an integral operator. The kernel of this operator includes a factor formed from the fundamental solution of the linear delay differential equation. Although the properties of the fundamental solutions are known, in general there is no closed form for the fundamental solution. This paper describes a collocation procedure to approximate the fundamental solution before the integral operator is discretized. Using arguments on collectively compact operators, the eigenvalues of the discretized monodromy operator are shown to converge to the eigenvalues of the monodromy operator in integral form. The eigenvalues of the monodromy operator tell the stability of the linear delay differential equation. An application to several cases of the Van der Pol oscillator with delay will be given.

1. Introduction. Delay differential equations have occurred in many fields from biology [**37**] to population dynamics [**34**] to machine tool dynamics [**11, 38, 54**]. The study of machine tool dynamics has led to many problems involving delay differential equations. For example, in turning operations a cutting tool passes over a workpiece many times successively. The forces on the tool depend on chip thickness which is dependent on the tool's current position and its position one previous revolution of the workpiece, thus introducing a delay effect. Any irregularities in a previous cut produced by the tool can affect the current cut. The delay effect of the irregularities can introduce self-sustained oscillations of the tool against the workpiece, called *regenerative chatter*. This phenomenon has been studied by Tlusty and Polacek [**48**] and Tobias [**49**] as early as the 1960's. Mathematically,

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