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HOW TO SOLVE HAMMERSTEIN EQUATIONS

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To Ken, with friendship and admiration

What is a good method for finding solutions $u : \Omega \to \mathbf{R}$ *of the nonlinear integral equation of Hammerstein type*

$$u(x) = v(x) + \lambda \int_\Omega k(x,y) f(y,u(y))\,dy, \quad \lambda \in \mathbf{R} \tag{1}$$

with given functions $v : \Omega \to \mathbf{R}$, $k : \Omega \times \Omega \to \mathbf{R}$, *and* $f : \Omega \times \mathbf{R} \to \mathbf{R}$ *?* This is the question which provides the main focus of this brief note. More precisely, we will discuss a variety of methods (topological degree, fixed point methods, spectral theory, variational approach, monotonicity methods, positivity methods, etc.) which turn out to be useful tools for solving (1). We point out that the presentation is *quite elementary*, so this note may be considered as a stimulation for exercises for students attending courses in nonlinear analysis, operator theory, or integral equations, rather than a sophisticated research contribution.

Usually, equation (1) is written as an operator equation

$$u - \lambda A u = v, \tag{2}$$

where the Hammerstein operator A may be represented as composition $A = KF$ of the linear Fredholm operator

$$Ku(x) = \int_\Omega k(x,y) u(y)\,dy \tag{3}$$

generated by the kernel function k, and the nonlinear Nemytskij operator

$$Fu(x) = f(x,u(x)) \tag{4}$$

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