

**NONCONFORMING FINITE ELEMENT  
METHODS FOR A CLAMPED PLATE  
WITH ELASTIC UNILATERAL OBSTACLE**

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Dedicated to Professor Kendall Atkinson

It is a privilege being his colleague and collaborator.

**ABSTRACT.** In this paper we analyze nonconforming finite element methods for solving a fourth order boundary value problem describing the deformation of a clamped elastic thin plate unilaterally constrained by an elastic obstacle. Optimal order error estimates are derived for both continuous and discontinuous nonconforming finite elements.

**1. Introduction.** Nonconforming methods are popularly employed in solving high order differential equations. For fourth order boundary value problems, conforming finite elements require  $C^1$  continuity. For multi-dimensional spatial domains, it is not easy to construct  $C^1$  elements, and the resulting  $C^1$  elements are usually difficult to use. On the contrary, nonconforming finite elements are easy to construct and easy to use, since the smoothness requirement on finite element functions is weakened to either  $C^0$  continuity or even less than  $C^0$  continuity. Application of nonconforming finite element methods is not limited to fourth order or higher order problems, though; they offer more efficient solution algorithms for numerous other problems, see, e.g., [3, p. 208].

It is more delicate to provide convergence and error analysis for nonconforming finite element methods than for conforming finite element methods. An early reference on the mathematical analysis of nonconforming finite element methods for the plate bending problem is [9]. A patch test was proposed and is widely used by engineers for

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