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REGULARIZATION OF FIRST KIND INTEGRAL EQUATIONS WITH APPLICATION TO COUETTE VISCOMETRY

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ABSTRACT. The recovery of flow curves for non-Newtonian fluids from Couette rheometry measurements involves the solution of a quite simple first kind Volterra integral equation with a discontinuous kernel. In this paper, a new implementation of regularization is proposed. It involves the direct regularization of the observational equations through the construction of basis functions that exploit the mathematical structure in the integral equation. The proposed implementation is first derived for a general first kind integral equation and then applied to the Couette rheometer equation. For the regularization of this problem, the basis functions take on a form similar to that for B-splines.

1. Introduction. From a rheological as well as an integral equation and numerical analysis perspective, the recovery of flow curves for non-Newtonian fluids from torque measurements on a Couette (coaxial cylindrical) rheometer has a long and interesting history which dates from Couette's (1890) invention, [**21**]. Rheologically, because it is a simple and fairly inexpensive experiment to perform, it is still a widely utilized procedure to characterize the viscometric properties of Newtonian and non-Newtonian fluids. Furthermore, when compared with capillary and cone-and-plate rheometry, it has a number of important advantages including ease of construction and alignment, experimental accuracy and predictability of secondary flows. Exact solutions are only known for the inversion of capillary and cone-and-plate measurements, and this is one of the reasons behind their historic popularity, especially before the advent of electronic computers.

However, the importance of Couette rheometers relates more to their industrial rather than their scientific use. They allow an assessment of a non-Newtonian fluid to be made relatively quickly and inexpensively. A snapshot of their wide range of industrial application can be

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