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ON INTEGRAL EQUATION FORMULATIONS OF A CLASS OF EVOLUTIONARY EQUATIONS WITH TIME-LAG

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Dedicated to Kendall Atkinson

ABSTRACT. In discussions of certain neutral delay differential equations in Hale's form, the relationship of the original problem with an integrated form (an integral equation) proves to be helpful in considering existence and uniqueness of a solution and sensitivity to initial data. Although the theory is generally based on the assumption that a solution is continuous, natural solutions of neutral delay differential equations of the type considered may be discontinuous. This difficulty is resolved by relating the discontinuous solution to its restrictions on appropriate (half-open) subintervals where they are continuous and can be regarded as solutions of related integral equations. Existence and unicity theories then follow. Furthermore, it is seen that the discontinuous solutions can be regarded as solutions in the sense of Carathéodory (where this concept is adapted from the theory of ordinary differential equations, recast as integral equations).

1. The forms of integral equation considered. The *integral equations* discussed in this paper are in the form

$$y(t) = g\big(t, y(t), y(t-\tau(t))\big) + \int_{t_0}^{t} f\big(s, y(s), y(s-\tau(s))\big)\mathrm{d}s + z^0 \tag{1.1}$$

or the form

$$y(t) = \gamma\Big(t, y(t-\tau(t)), \int_{t_0}^{t} f\big(s, y(s), y(s-\tau(s))\big)\mathrm{d}s + z^0\Big). \tag{1.2}$$

In either case, the equation holds for $t \in \mathfrak{I}_0$ where $\mathfrak{I}_0$ is $[t_0, T]$ or $[t_0, T)$ (for $t_0 < T \in \mathbf{R} \cup \infty$), and $y(t)$ is prescribed on a suitable initial interval $[t_{-1}, t_0] \subset (-\infty, t_0]$. The situations considered can

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