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ON THE REGULARITY OF SOLUTIONS TO VOLTERRA FUNCTIONAL INTEGRO-DIFFERENTIAL EQUATIONS WITH WEAKLY SINGULAR KERNELS

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Dedicated to Professor Ken Atkinson

ABSTRACT. We study the regularity properties of solutions for various classes of Volterra functional integro-differential equations with nonvanishing delays and weakly singular kernels. In particular, we characterize equations in which "supersmoothing," smoothing, or no smoothing occurs at the primary discontinuity points induced by the nonvanishing delay. These results will play a crucial role in the design and analysis of methods for the numerical solution of such functional equations with nonsmooth solutions.

1. Introduction. In this paper we analyze the regularity properties of solutions to Volterra functional integro-differential equations (VFIDEs) with weakly singular kernels and containing a delay function $\theta = \theta(t) := t - \tau(t)$ satisfying the following conditions (D1)–(D3) on a given (compact) interval $J := [t_0, T]$:

(D1) $\tau \in C^d(J)$ for some $d \geq 0$;

(D2) $\tau(t) \geq \tau_0 > 0$ for all $t \in J$;

(D3) θ is strictly increasing on J.

The points $\{\xi_\mu\}$ induced by (D2) and defined implicitly by the recursion

$$\theta(\xi_\mu) = \xi_\mu - \tau(\xi_\mu) = \xi_{\mu-1} \quad (\mu = 1, \dots), \quad \text{with} \quad \xi_0 := t_0, \tag{1.1}$$

are called the primary discontinuity points corresponding to θ; they play a crucial role in the subsequent regularity analysis. Note that

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