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QUALOCATION FOR BOUNDARY INTEGRAL EQUATIONS USING SPLINES WITH MULTIPLE KNOTS

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Dedicated to Kendall Atkinson

ABSTRACT. This paper provides an analysis of the qualocation method for periodic pseudodifferential operators, with multiple knot splines used for the trial and test spaces. A recently introduced basis for the multiple knot periodic splines leads to a relatively easy analysis. Convergence is proved with the aid of approximation properties of the qualocation projection and of inverse stability estimates that are characterized by necessary and sufficient algebraic conditions. The analysis of the variable coefficient case uses a local principle and recently established commutator properties.

1. Introduction. In this paper we study the qualocation method for pseudodifferential operators of the form

$$L = L_0 + L_1, \tag{1.1}$$

where

$$L_0 v(x) := \sum_{n=-\infty}^{\infty} \sigma_0(x,n)\, \hat{v}(n) e^{i2\pi nx} \quad \text{for} \quad x \in \mathbf{T}, \tag{1.2}$$

and L_1 is a suitable perturbation (see below). Here $\mathbf{T} := \mathbf{R}\backslash\mathbf{Z}$ is the one-dimensional torus of length 1, and

$$\hat{v}(n) := \int_{\mathbf{T}} v(x) e^{-i2\pi nx}\, dx \quad \text{for} \quad n \in \mathbf{Z}$$

are the complex Fourier coefficients of a 1-periodic distribution $v : \mathbf{T} \to \mathbf{R}$, so that

$$v(x) = \sum_{n=-\infty}^{\infty} \hat{v}(n) e^{i2\pi nx} \quad \text{for} \quad x \in \mathbf{T}.$$

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