

**SUPERCONVERGENCE OF THE ITERATED
HYBRID COLLOCATION METHOD
FOR WEAKLY SINGULAR
VOLTERRA INTEGRAL EQUATIONS**

MIN HUANG AND YUESHENG XU

Dedicated to Professor K.E. Atkinson on the occasion
of his 65th birthday with friendship and esteem

ABSTRACT. A hybrid collocation method for Volterra integral equations with weakly singular kernels was introduced in [12]. The main purpose of this paper is to study *superconvergence* properties of the *iterated* hybrid collocation solution. It is proved that the iterated collocation solution has an improvement on order of convergence for the original collocation solution provided that suitable collocation parameters are chosen. Moreover, we apply the hybrid collocation and the associated iterated method to solving Volterra integro-differential equations with weakly singular kernels. Numerical examples are presented to confirm the superconvergence results of the iterated collocation methods.

1. Introduction. The main purpose of this paper is to study superconvergence properties of iterated solutions of Volterra integral equations of the second kind with weakly singular kernels, based on the hybrid collocation method developed in [12].

It is well known that the solution of the equations exhibits a singularity near the left end-point of the domain because of the weak singularity in the kernel. When we develop a numerical method for solving equations of this type we should take this into account in order to produce an approximate solution with high order convergence. The collocation

2000 AMS *Mathematics Subject Classification.* Primary 65R20, 45D05.

Key words and phrases. Volterra integral equations, Volterra integro-differential equations, weakly singular kernels, iterated hybrid collocation methods, superconvergence.

Supported in part by the US Natl. Sci. Foundation under grants CCR-0407476, by the Natural Sci. Foundation of China under grant 10371122 and by the Ministry of Educ., P.R. China, under the Changjiang Scholar Chair Professorship program.

The second author is the corresponding author.

Received by the editors on Aug. 19, 2005, and in revised form on Dec. 31, 2005.