JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 18, Number 1, Spring 2006

COMPACTNESS ESTIMATES FOR INTEGRAL OPERATORS OF VECTOR FUNCTIONS WITH NONMEASURABLE KERNELS

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Dedicated to Kendall Atkinson

ABSTRACT. Estimates for the measure of noncompactness of integral operators of vector functions are proved. In particular, for linear integral operators of vector functions with nonmeasurable kernels a Mönch type compactness result is obtained.

1. Introduction. For scalar functions, it is well known that the Urysohn integral operator

$$(1) \qquad Ax(t):=\int_0^1 g(t,s,x(s))\,ds, \quad t\in[0,1]$$

is usually compact. In particular, A is compact in the space $C([0,1])$ under mild continuity assumptions on g. This is a consequence of the Arzelá-Ascoli theorem, because the image of bounded sets has equicontinuous norm (provided that, e.g., g is continuous).

Moreover, if g is a Carathéodory function, i.e., $g(\,\cdot\,,\,\cdot\,,u)$ is (strongly) (Bochner) measurable for each u, and $g(t,s,\,\cdot\,)$ is continuous for almost all $(t,s)\in[0,1]^2$, then, under some growth assumptions on g, the operator A maps compactly into the spaces $L_p([0,1])$, $1\le p<\infty$, or, more generally, into regular ideal spaces. These are classical results of Krasnosel'skiĭ[**9**] and Zabreĭko, see e.g., [**9, 10, 28**], and it is also possible to weaken the growth conditions slightly [**16, 20**].

We are interested in the case that the functions x and g assume values in Banach (or at least normed) spaces U and V, respectively, where the

Received by the editors on May 6, 2005.
Key words and phrases. Integral operator, kernel operator, Urysohn operator, measure of noncompactness, ideal space, spaces of measurable functions.
The paper was written in the framework of a Heisenberg Fellowship (Az. VA 206/1-1 and VA 206/1-2). Financial support by the DFG is gratefully acknowledged.