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EXACT SOLUTION OF SOME INTEGRAL EQUATIONS OVER A CIRCULAR DISC

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ABSTRACT. Two-dimensional integral equations of the first kind over a circular disc are considered. The kernels involve the distance between two points on the disc raised to an arbitrary power. A review is given, comparing several published exact solutions for weakly-singular equations: these solutions are complicated, but three of them are shown to be equivalent. Some extensions to hypersingular equations are discussed.

1. Introduction. This paper is concerned with integral equations of the form

$$\int_D \frac{w(x,y)}{R^{2\alpha}}\,dx\,dy = p(x_0,y_0), \quad (x_0,y_0)\in D. \tag{1}$$

Here, $D=\{(x,y): x^2+y^2<a^2\}$ is a circular disc of radius a, centered at the origin in the xy-plane, p is a given function and w is to be found. R is the distance between two points in the disc,

$$R=\{(x-x_0)^2+(y-y_0)^2\}^{1/2},$$

and α is a positive parameter. The kernel $R^{-2\alpha}$ is weakly singular for $0<\alpha<1$, and it is hypersingular for $\alpha\geq 1$. (We shall define "hypersingular" later. Note that $\alpha=1$ does not lead to a "singular" integral equation, as the principal-value integral of R^{-2} does not exist.)

The case $\alpha = 1/2$ is classical: it arises in the problem of the electrified disc [**6, 24, 27**]. This problem requires the determination of a harmonic function in three-dimensional space, $V(x,y,z)$, with the Dirichlet condition, $V=1$, on the disc and the condition $V\to 0$ at infinity.

More generally, the weakly-singular case ($0<\alpha<1$) has been studied by several authors. Complicated formulas for the exact solution of (1)

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