JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 17, Number 4, Winter 2005

NONLINEAR INTEGRAL EQUATIONS WITH INCREASING OPERATORS IN MEASURE SPACES

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ABSTRACT. In this paper we consider a class of integral equations in measure spaces. Remarkable and important special integral equations are contained among them, which have been extensively investigated nowadays. The main results of this paper are existence theorems for the studied integral equations under the condition that the operator defined by the equation is increasing. Moreover, there are some auxiliary results which are interesting in their own rights. We shall see that some of the problems formulated for the classical integral equations can be solved in a very satisfactory way in this essentially more general case, and the results give unified approaches of the problems. Finally, some applications are given.

1. Introduction. In what follows $(X, \mathcal{A}, \mu_i)$, $i = 1, \dots, n$ are measure spaces, S is a function from X into $\mathcal{A}$, and $\mu := \sum_{i=1}^{n} \mu_i$.

In this paper we study integral equations of the form

$$y(x) = f(x) + \sum_{i=1}^{n} g_i(x) \int_{S(x)} h_i \circ y \, d\mu_i, \tag{1}$$

where $f : D_f (\subset X) \to \mathbf{R}$, $g_i : D_{g_i} (\subset X) \to \mathbf{R}$, $i = 1, \dots, n$, and $h_i : I_i$ $(\subset \mathbf{R}) \to \mathbf{R}$ for $i = 1, \dots, n$.

We recall some concepts from measure theory that will be used in the present work. When we consider a measure, we take it as understood that its domain is a σ-algebra. The integrable functions, with respect to a measure, over a measurable set are regarded as almost measurable on this set. The product of finitely many measure spaces is understood as in [**8**].

2000 AMS *Mathematics Subject Classification.* Primary 45G10, Secondary 47B60.
Key words and phrases. Integral equation, measure space, increasing operator.
Received by the editors on March 22, 2005, and in revised form on September 30, 2005.