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ALMOST AUTOMORPHIC MILD SOLUTIONS TO SOME SEMI-LINEAR ABSTRACT DIFFERENTIAL EQUATIONS WITH DEVIATED ARGUMENT

CIPRIAN G. GAL

ABSTRACT. In this paper we consider the semi-linear differential equation with deviated argument $x'(t) = Ax(t) + f(t, x(t), x[\alpha(x(t), t)])$, $t \in \mathbf{R}$, in a Banach space $(X, \|\cdot\|)$, where A is the infinitesimal generator of a C_0-semigroup satisfying some conditions of exponential stability. Under suitable conditions on the functions f and α we prove the existence and uniqueness of an almost automorphic mild solution to the equation.

1. Introduction. Everywhere in the paper, $(X, \|\cdot\|)$ will be a Banach space.

The concept of almost automorphy is a generalization of almost periodicity and it has been introduced in the literature by Bochner, as follows.

Definition 1.1. We say that a continuous function $f : \mathbf{R} \to X$, is almost automorphic, if every sequence of real numbers $(r_n)_n$, contains a subsequence $(s_n)_n$, such that for each $t \in \mathbf{R}$, there exists $g(t) \in X$ with the property

$$\lim_{n\to+\infty} d(g(t), f(t+s_n)) = \lim_{n\to+\infty} d(g(t-s_n), f(t)) = 0.$$

(The above convergence on $\mathbf{R}$ is pointwise.) The set of all almost automorphic functions with values in X is denoted by $AA(X)$.

In a very recent paper [**3**] the existence and uniqueness of almost automorphic mild solutions with values in Banach spaces, for the

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