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ITERATIVELY REGULARIZED GRADIENT METHOD WITH A POSTERIORI STOPPING RULE FOR 2D INVERSE GRAVIMETRY PROBLEM

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ABSTRACT. A nonlinear operator equation $F(x) = f_\delta$, $\|f - f_\delta\| \leq \delta$, on a pair of real Hilbert spaces H_1 and H_2 is considered. The operator F is assumed to be Fréchet differentiable without such structural assumptions as monotonicity, invertibility of $F'(x)$, etc., i.e. the problem is ill-posed. In order to solve the above equation numerically we suggest the iteratively regularized gradient method [**3**] combined with a new generalized discrepancy principle:

$$\|F(x_N) - f_\delta\|^2 < \tau\delta \leq \|F(x_n) - f_\delta\|^2, \quad 0 \leq n < N,\ \tau > 1.$$

A convergence theorem is proven under a source type condition

$$\hat{x} - x_0 = F'^*(\hat{x})v, \quad v \in H_2.$$

The proposed algorithm is tested on the 2D inverse gravimetry problem [**15**] reduced to a nonlinear integral equation of the first kind. The results of numerical simulations are presented and some practical recommendations on the choice of parameters are given.

1. Introduction. Consider the following inverse problem:

$$F(x) = f, \quad F : H_1 \longrightarrow H_2, \tag{1.1}$$

where F is a nonlinear Fréchet differentiable operator on a pair of real Hilbert spaces H_1 and H_2. Assume that the element $f \in H_2$ is given by its δ-approximation:

$$\|f - f_\delta\| \leq \delta \tag{1.2}$$

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