

MARCINKIEWICZ FUNCTIONS ALONG FLAT SURFACES WITH HARDY SPACE KERNELS

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ABSTRACT. In this paper, we study Marcinkiewicz integral operators along subvarieties determined by flat surfaces with kernels in the Hardy space $\mathbf{H}^1(\mathbf{S}^{n-1})$. We establish the L^p boundedness of our operators under weak convexity assumptions on the surfaces. Moreover, we establish the L^p boundedness of the corresponding Marcinkiewicz integral operators that are related to area integral and Littlewood-Paley g_λ^* functions. Our results offer substantial improvements of previously known results.

1. Introduction and statement of results. Let \mathbf{R}^n , $n \geq 2$, be the n -dimensional Euclidean space and \mathbf{S}^{n-1} the unit sphere in \mathbf{R}^n equipped with the induced Lebesgue measure $d\sigma$. Let Ω be a homogeneous function of degree zero on \mathbf{R}^n that is integrable on \mathbf{S}^{n-1} and satisfies

$$(1.1) \quad \int_{\mathbf{S}^{n-1}} \Omega(y) d\sigma(y) = 0.$$

For a suitable mapping $\Phi : \mathbf{R}^n \rightarrow \mathbf{R}^n$, consider the Marcinkiewicz integral operator

$$(1.2) \quad \mu_{\Omega, \Phi} f(x) = \left(\int_{-\infty}^{\infty} |F_{\Omega, \Phi, t}(x)|^2 2^{-2t} dt \right)^{1/2},$$

where

$$(1.3) \quad F_{\Omega, \Phi, t}(x) = \int_{|y| \leq 2^t} f(x - \Phi(y)) |y|^{1-n} \Omega(y) dy.$$

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