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A CLASS OF MAXIMAL OPERATORS RELATED TO ROUGH SINGULAR INTEGRALS ON PRODUCT SPACES

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ABSTRACT. This paper is concerned with studying the L^p boundedness of a class of maximal operators $\mathcal{S}_\Omega^{(\gamma)}$ related to rough singular integrals on product spaces. We obtain appropriate L^p bounds for such maximal operators and establish the optimality of our condition on the kernel for the L^2 boundedness of $\mathcal{S}_\Omega^{(2)}$. Our results improve substantially the main result obtained by Ding in [**8**].

1. Introduction and statement of results. Throughout this paper, we let ξ' denote $\xi/|\xi|$ for $\xi \in \mathbf{R}^n \backslash \{0\}$ and p' denote the exponent conjugate to p, that is, $1/p+1/p'=1$. Let $n, m \geq 2$. Suppose that $\mathbf{S}^{d-1}$ ($d = n$ or m) is the unit sphere of $\mathbf{R}^d$ equipped with the normalized Lebesgue measure $d\sigma = d\sigma(x')$.

In [**7**], Chen and Lin studied the L^p boundedness of a class of maximal operators $\mathcal{M}_\Omega^{(\gamma)}$ defined by

$$\mathcal{M}_\Omega^{(\gamma)} f(x) = \sup_h \left| \int_{\mathbf{R}^n} f(x-y) h(|y|)\, \Omega\,(y/|y|)\, |y|^{-n}\, dy \right|,$$

where the supremum is taken over the set $\{h : \|h\|_{L^\gamma(\mathbf{R}^+, dr/r)} \leq 1\}$, $\gamma > 1$ and $\Omega \in L^1(\mathbf{S}^{n-1})$ is a function satisfying the cancelation condition

$$\int_{\mathbf{S}^{n-1}} \Omega(y')\, d\sigma(y') = 0. \tag{1.1}$$

Chen and Lin in [**7**] proved the L^p boundedness of the maximal operator $\mathcal{M}_\Omega^{(\gamma)}$ under a smoothness condition on Ω as described in the following theorem:

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