

## A FAST WAVELET COLLOCATION METHOD FOR INTEGRAL EQUATIONS ON POLYGONS

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Dedicated to Professor Kendall E. Atkinson on the occasion  
of his 65th birthday with friendship and esteem

**ABSTRACT.** We develop a fast wavelet collocation method for solving Fredholm integral equations of the second kind with weakly singular kernels on *polygons*, following the general setting introduced in a recent paper [11]. Specifically, we construct wavelet functions and *multiscale* collocation functionals having vanishing moments on polygons. Critical issues for numerical implementation of this method are considered, such as a practical matrix compression scheme, numerical integration of weakly singular integrals, error controls of numerical quadratures and numerical solutions of the resulting compressed linear systems. The estimate of the computational complexity ensures the proposed method is a fast algorithm. Numerical experiments are presented to demonstrate the proposed methods and confirm the theoretical estimates.

**1. Introduction.** In a recent paper [11], a general setting was proposed for fast wavelet collocation methods for solving Fredholm integral equations of the second kind with weakly singular kernels. The main purpose of this paper is to develop a concrete fast wavelet collocation method for the integral equations on polygons, following the general recipe given in [11]. Specifically, we construct piecewise linear wavelets on polygons and their corresponding collocation functionals, both having vanishing moments of order two. We also consider several critical issues for numerical implementation of this method, such as a practical matrix compression scheme, numerical integration of weakly singular integrals, error controls of numerical quadratures and fast numerical solutions of the resulting compressed linear systems.

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