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STABILITY IN LINEAR VOLTERRA INTEGRODIFFERENTIAL EQUATIONS WITH NONLINEAR PERTURBATION

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ABSTRACT. A Lyapunov functional is employed to obtain conditions that guarantee stability, uniform stability and uniform asymptotic stability of the zero solution of a scalar linear Volterra integrodifferential equation with nonlinear perturbation.

1. Introduction. In this paper we consider the scalar linear Volterra integrodifferential equation

$$x'(t) = h(t)x(t) + \int_0^t C(at - s)x(s)\,ds \tag{1.1}$$

and its perturbed form

$$x'(t) = h(t)x(t) + \int_0^t C(at - s)x(s)\,ds + g(t, x(t)) \tag{1.2}$$

where a is a constant, $a > 1$. The function $g(t, x(t))$ is continuous in t and x and satisfies $|g(t, x(t))| \leq \lambda(t)|x(t)|$, where $\lambda(t)$ is continuous. Moreover, $h(t)$ is continuous for all $t \geq 0$ and $C : \mathbf{R} \to \mathbf{R}$ is continuous. We study the stability properties of the zero solution of either (1.1) or (1.2) and we construct suitable Lyapunov functionals in the analysis.

We point out that if $C \in L^1[0, \infty)$, then the equations (1.1) and (1.2) become fading memory problems. When $a > 1$, the memory term $\int_0^t C(at - s)\,ds = \int_{(a-1)t}^{at} C(u)\,du$ tends to zero as $t \to \infty$, that is, the memory fades away completely. On the other hand, if $0 < a < 1$, the memory term never fades away completely; it tends to a constant as $t \to \infty$. For $a = 1$, equations (1.1) and (1.2) are the well-known convolution equations. Many researchers have studied stability

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