JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 17, Number 3, Fall 2005

THE p-VERSION OF THE BOUNDARY ELEMENT METHOD FOR A THREE-DIMENSIONAL CRACK PROBLEM

ALEXEI BESPALOV AND NORBERT HEUER

ABSTRACT. We study the p-version of the boundary element method for a crack problem in linear elasticity with Dirichlet boundary conditions. The unknown jump of the traction has strong edge singularities and is approximated by solving an integral equation of the first kind with weakly singular operator. We prove a quasi-optimal a priori error estimate in the energy norm. For sufficiently smooth given data, this gives a convergence like $cp^{-1+\varepsilon}$ with $\varepsilon > 0$. Here, p denotes the polynomial degree of the piecewise polynomial functions used to approximate the unknown.

1. Introduction and formulation of the problem. We analyze the convergence of the p-version of the boundary element method (BEM) with weakly singular integral operator for problems in $\mathbf{R}^3$. That is, we study approximation properties of piecewise polynomial functions on surfaces in a negative order Sobolev space (order $-1/2$).

The p-version uses piecewise polynomial spaces on fixed meshes and increases the polynomial degrees, whereas the more conventional h-version improves approximations by mesh refinement and by using piecewise polynomials of lower, fixed degrees. It is well known that an appropriate combination of mesh refinement and polynomial degree distribution (the hp-version with geometrically graded meshes) may lead to an exponential rate of convergence, even in the presence of singularities (for the FEM, see [**9, 10**] and for the BEM we refer to [**13–15, 17**]). The approximation strategy of such hp-methods is to use polynomial degrees of lowest order where solutions behave singularly and to use high order polynomials where solutions are smooth. With

2000 AMS *Mathematics Subject Classification.* 41A10, 65N15, 65N38.
Key words and phrases. p-version, boundary element method, linear elasticity, singularities.
Work of the first author financed by the FONDAP Program in Appl. Mathematics, Chile. Work of the second author supported in part by the FONDAP Program in Appl. Mathematics and Fondecyt project no. 1040615, both Chile.
Received by the editors on July 20, 2005.

Copyright ©2005 Rocky Mountain Mathematics Consortium