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BOOK REVIEW

Collocation Methods for Volterra Integral and Related Functional Equations, by Hermann Brunner,
Cambridge Monographs on Applied and Computational Mathematics,
vol. 15, Cambridge University Press, Cambridge, U.K.
$120. xiv + 597 pp., hardcover. ISBN 0-521-80615-1.

By Volterra Functional Equations is usually meant a broad class of time-dependent equations with memory terms, that is, equations whre, at any time t, the unknown function $y(t)$ depends on some (possibly infinitely many) past values of the function y itself. This class includes, as special instances, integral equations and integro-differential equations of first and second kind, delay differential equations, integral and integro-differential equations with delay, delay equations with neutral terms, etc.

The properties of the kernel (regular, weakly singular, of convolution type, etc.) in the integral equations and the particular form of the delay function (constant, time dependent, state dependent, vanishing, proportional, unbounded, etc.) give rise to a variety of equations that require specific approaches but share the need of continuous approximations for their numerical integration. Consequently, "collocation" and, in particular, "piecewise polynomial collocation" methods turn out to provide the most natural approach for integrating such a class of equations.

On the basis of this fact the author, who, since the early seventies, has made significant theoretical and numerical contributions to this area, provides in this expository research monograph a well-balanced combination of the basic knowledge about several functional equations and the extensive analysis of the collocation method for their discretization. The author begins the book by introducing the collocation method applied to initial value problems for ODEs and then uses it throughout the book as leit-motif for deriving the discrete analogue of several Volterra Functional Equations and for outlining the intrinsic difficulties in their numerical integration inherited by the presence of the memory terms.