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SOME VOLTERRA-TYPE FRACTIONAL INTEGRO-DIFFERENTIAL EQUATIONS WITH A MULTIVARIABLE CONFLUENT HYPERGEOMETRIC FUNCTION AS THEIR KERNEL

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ABSTRACT. Motivated essentially by several recent works on interesting generalizations of the first-order Volterra-type integro-differential equation governing the unsaturated behavior of the free electron laser (FEL) by making use of fractional calculus, that is, calculus of integrals and derivatives of an arbitrary real or complex order, the authors investigate the solutions of several Cauchy-type and Cauchy problems associated with some general fractional Volterra-type integro-differential equations in which the kernel involves the confluent hypergeometric function $\Phi_2^{(n)}$ in n complex variables. The closed-form solution of each of these general Cauchy-type problems is derived in terms of the function $\Phi_2^{(n)}$ itself. Several special cases of the main results are also shown to yield generalizations of the results investigated in the aforementioned and other earlier works.

1. Introduction. The unsaturated behavior of the free electron laser (FEL), when no field mode structures are taken into consideration, is governed by the following first-order integro-differential equation of Volterra type, cf. [**6, 8**]:

$$\frac{d}{d\tau}\,h(\tau) = -i\pi g_0 \int_0^\tau \xi \exp(i\nu\xi)\, h(\tau-\xi)\, d\xi, \tag{1.1}$$

where τ is a dimensionless time variable ($0 \leqq \tau \leqq 1$), g_0 is a positive constant called the small signal gain, and ν is a real constant referred

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