JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 17, Number 2, Summer 2005

ON THE ANALYTICAL SOLUTIONS OF TWO SINGULAR INTEGRAL EQUATIONS WITH HILBERT KERNELS

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ABSTRACT. The analytical solution of two singular integral equations with Hilbert kernel of the first and second kind, respectively, is derived from the known solution of the corresponding singular integral equations with Cauchy kernel of the first and second kind by introducing a proper change of variables.

1. Introduction. The representation of all solutions of the singular integral equations

$$\int_{-1}^{+1} \frac{\tilde{f}(x)}{y-x}\,dx = \tilde{g}(y), \quad -1 < y < 1 \tag{1.1}$$

$$\tilde{f}(y) - \frac{i\lambda}{\pi}\int_{-1}^{+1} \frac{\tilde{f}(x)}{y-x}\,dx = \tilde{g}(y), \quad -1 < y < 1 \tag{1.2}$$

where λ is real and the integrals are defined in the Cauchy principal value sense, is of key importance in many applications. Among the several authors who have examined this problem, Söhngen, [**13, 14**], and Tricomi, [**7, 16, 17**], appear to be the first to have obtained fundamental results on this topic. Here we recall some of them.

Theorem 1 (see [**13, 14, 16**]). *If in* (1.1) $\tilde{g} \in L_p$, $p > 1$, *then* $\tilde{f} \in L_q$ *for some* $q > 1$ *and necessarily has the form*

$$\tilde{f}(y) = -\frac{1}{\pi^2}\frac{1}{\sqrt{1-y^2}}\int_{-1}^{+1}\frac{\tilde{g}(x)\sqrt{1-x^2}}{y-x}\,dx + \frac{C}{\sqrt{1-y^2}} \tag{1.3}$$

where C *is an arbitrary constant.*

Key words and phrases. Singular integral equations, first and second kind, Hilbert kernel, analytical solution.
This work has been supported by the Ministero dell'Istruzione, dell'Università e della Ricerca of Italy.
Received by the editors on Feb. 1, 2005, and in revised form on April 21, 2005.