

STEADY STATE OSCILLATION PROBLEMS IN THE THEORY OF ELASTICITY FOR CHIRAL MATERIALS

DAVID NATROSHVILI, LEVAN GIORGASHVILI
AND SHOTA ZAZASHVILI

Dedicated to the memory of Professor Ilia Vekua

ABSTRACT. Mathematical problems of the theory of steady state oscillations of hemitropic (chiral) elastic materials are considered. In the case of unbounded domains, the generalized Sommerfeld-Kupradze type radiation conditions are introduced and in the space of radiating solutions the uniqueness results are established. Applying the potential method and the theory of pseudodifferential equations, the unique solvability in various function spaces of the Dirichlet, Neumann and mixed boundary value problems for the steady state oscillation equations are proved. Regularity properties and representability of solutions by layer potentials are analyzed in the cases of smooth and Lipschitz domains.

1. Introduction. A solid which is not isotropic with respect to rotation is called *noncentrosymmetric*, *acentric*, *hemitropic*, or *chiral*. Materials may exhibit chirality on the atomic scale, as in quartz and in biological molecules, as well as on a large scale, as in composites with helical or screw-shaped inclusions (for details see, e.g., [1, 20 and the references therein]).

Mathematical models describing the chiral properties of elastic materials have been proposed by Aero and Kuvshinski [1,2] (for the history of the problem see also [27, 36, 38, 46 and the references therein]).

Particular problems of the elasticity theory of hemitropic continuum related to the present paper have been considered in [10, 20, 22–24, 36, 37, 39, 46]. In [33, 34] the basic boundary value problems

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