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BOUNDEDNESS IN NONLINEAR FUNCTIONAL DIFFERENTIAL EQUATIONS WITH APPLICATIONS TO VOLTERRA INTEGRODIFFERENTIAL EQUATIONS

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ABSTRACT. Non-negative definite Lyapunov functions are employed to obtain sufficient conditions that guarantee boundedness of solutions of nonlinear functional differential systems. The theory is illustrated with several examples.

1. Introduction. In this paper, we make use of non-negative definite Lyapunov functions and obtain sufficient conditions that guarantee the boundedness of all solutions of the system of functional differential equations

$$x'(t) = G(t, x(s);\ 0 \le s \le t) \overset{\text{def}}{=} G(t, x(\cdot)) \tag{1.1}$$

where $x \in \mathbf{R}^n$, $G : \mathbf{R}^+ \times \mathbf{R}^n \to \mathbf{R}^n$ is a given nonlinear continuous function in t and x. For a vector $x \in \mathbf{R}^n$ we take $\|x\|$ to be the Euclidean norm of x. Let $t_0 \ge 0$, then for each continuous function $\phi : [0, t_0] \to \mathbf{R}^n$, there is at least one continuous function $x(t) = x(t, t_0, \phi)$ on an interval $[t_0, I]$ satisfying (1.1) for $t_0 \le t \le I$ and such that $x(t, t_0, \phi) = \phi(t)$ for $0 \le t_0 \le I$. It is assumed that at $t = t_0$, $x'(t)$ is the right hand derivative of $x(t)$. For conditions ensuring existence, uniqueness and continuability of solutions of (1.1), we refer the reader to [**2**] and [**5**].

In [**10**], the author studied the boundedness of solutions of the initial value problem

$$x'(t) = G(t, x(t)); \quad t \ge 0$$
$$x(t_0) = x_0$$

by making use of non-negative definite Lyapunov functions.

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