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APPROXIMATE SOLUTION OF MULTIVARIABLE INTEGRAL EQUATIONS OF THE SECOND KIND

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ABSTRACT. For a multidimensional integral equation of the second kind with a smooth kernel, using the orthogonal projection onto a space of discontinuous piecewise polynomials of degree r, Atkinson has established an order $r + 1$ convergence for the Galerkin solution and an order $2r+2$ convergence for the iterated Galerkin solution. In a recent paper [**15**], a new method based on projections has been shown to give a $4r + 4$ convergence for one-dimensional second kind integral equations. The size of the system of equations that must be solved in implementing this method remains the same as for the Galerkin method. In this paper, this method is extended to multi-dimensional second kind equations and is shown to have convergence of order $4r + 4$. For interpolatory projections onto a space of piecewise polynomials, it is shown that the order of convergence of the new method improves on the previously established orders of convergence for the collocation and the iterated collocation methods. A two-grid norm convergent method based on the new method is also defined.

1. Introduction. Over the years approximate solution of one dimensional Fredholm integral equations of the second kind has been extensively studied. (See [**2, 5, 7, 8, 11, 14, 20**].) The classical methods are the Galerkin method based on a sequence of projections converging pointwise to the identity operator and the Nyström method based on a numerical quadrature. The improvement of the Galerkin solution by using an iteration technique was first proposed by Sloan in [**18**]. Chandler [**7**], in his Thesis, proved that if the kernel and the righthand side are smooth, then, in the case of the orthogonal projection onto a space of piecewise polynomials, the order of convergence in the iterated Galerkin solution is twice that of the Galerkin solution. Chatelin and Lebbar [**9**] proved similar results for the iterated collocation at Gauss points. For situations where the righthand side of the operator equation is less smooth than the kernel of the integral operator, the higher

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