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REAL ANALYTIC DEPENDENCE OF SIMPLE AND DOUBLE LAYER POTENTIALS UPON PERTURBATION OF THE SUPPORT AND OF THE DENSITY

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ABSTRACT. We consider a hypersurface in Euclidean space $\mathbf{R}^n$ parametrized by a diffeomorphism of the unit sphere to $\mathbf{R}^n$, and a density function on the hypersurface, which we think as points in suitable Schauder spaces, and we consider the dependence of the corresponding simple and double layer potentials, which we also think as points in suitable Schauder spaces, upon variation of the diffeomorphism and of the density, and we show a real analyticity theorem for such dependence.

1. Introduction. In this paper, we plan to study the dependence of simple and of double layer potentials upon the hypersurface of integration, i.e., upon the support. We assume the hypersurface of integration to be of sphere-type. Namely, we denote by $\mathbf{B}_n \equiv \{x \in \mathbf{R}^n : |x| < 1\}$ the open unit ball in the Euclidean space $\mathbf{R}^n$, with $n \geq 2$, and we consider our hypersurface to be assigned by a diffeomorphism ϕ of $\partial\mathbf{B}_n$ onto $\phi(\partial\mathbf{B}_n) \subseteq \mathbf{R}^n$, such that $\phi(\partial\mathbf{B}_n)$ is an $(n-1)$-dimensional manifold imbedded in $\mathbf{R}^n$. Then we consider the set $\mathcal{A}_{\partial\mathbf{B}_n}$ of such admissible functions ϕ, see Lemma 2.5, and we consider the Schauder space $C^{m,\alpha}(\partial\mathbf{B}_n, \mathbf{R}^n)$. The set $C^{m,\alpha}(\partial\mathbf{B}_n, \mathbf{R}^n) \cap \mathcal{A}_{\partial\mathbf{B}_n}$ is open in $C^{m,\alpha}(\partial\mathbf{B}_n, \mathbf{R}^n)$, and we can think of ϕ as a point of such a set. If $f \in C^{m,\alpha}(\partial\mathbf{B}_n, \mathbf{R})$, then the function $f \circ \phi^{(-1)}$ is defined on $\phi(\partial\mathbf{B}_n)$, and it makes sense to consider the simple and double layer potentials

$$v[\phi, f](\xi) \equiv \int_{\phi(\partial\mathbf{B}_n)} S_n(\xi-\eta)\, f \circ \phi^{(-1)}(\eta)\, d\sigma_\eta \qquad \forall\, \xi \in \phi(\partial\mathbf{B}_n),$$

$$w[\phi, f](\xi) \equiv \int_{\phi(\partial\mathbf{B}_n)} \frac{\partial}{\partial\nu_\phi(\eta)} [S_n(\xi-\eta)]\, f \circ \phi^{(-1)}(\eta)\, d\sigma_\eta \quad \forall\, \xi \in \phi(\partial\mathbf{B}_n),$$

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