JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 16, Number 2, Summer 2004

ANALYSIS VIA INTEGRAL EQUATIONS OF AN IDENTIFICATION PROBLEM FOR DELAY DIFFERENTIAL EQUATIONS

CHRISTOPHER T.H. BAKER AND EVGENY I. PARMUZIN

ABSTRACT. We address the problem of determining the initial function $\varphi(t)$, for $t \in [-\tau, 0]$, given the solution $y(t) \equiv y(\varphi; t)$ of the linear delay differential equation

$$y'(t) - A(t)y(t) - B(t)y(t-\tau) = f(t), \quad t \in [0, T],$$

for which

$$y(t) = \varphi(t), \quad t \in [-\tau, 0].$$

The function $\varphi(t)$ is approximated by the function $\varphi_\star(t)$ that minimizes a certain parameter-dependent quadratic functional. The optimal function $\varphi_\star(t)$ is shown to satisfy a Fredholm integral equation, and the rôle of a regularization parameter is transparent from the form of this equation. (There is a related integral equation for $\varphi(t)$.) The convergence properties of an iterative method for finding $\varphi_\star(t)$, using an iteration that is based upon the delay equation for $y(t)$ and a corresponding adjoint equation, are established by considering an iteration for the solution of the Fredholm integral equation.

1. The nature of the problem. Studies have been undertaken, in the context of the mathematical modeling of biological data, see Remark 1.3, of the problem of determining a parametrized retarded differential equation, along with the corresponding initial function, such that the solution is a good fit to an observed function. Related problems have been addressed by others, e.g., [**7, 14**]. Our practical approach to answering the question relies upon the numerical solution of differential equations with deviating arguments, and the minimization of an objective function appearing in this paper; we discuss numerical experiments elsewhere. However, some interesting analysis involving integral equations arises in arriving at a theory for our technique; part of this material is presented below.

Here, we describe a method for determining an approximation to an initial function, given the solution to a linear delay differential equation

Received by the editors on May 7, 2004, and accepted on July 20, 2004.