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FAST BOOLEAN APPROXIMATION METHODS FOR SOLVING INTEGRAL EQUATIONS IN HIGH DIMENSIONS

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ABSTRACT. Solving integral equations in high dimensions requires a huge computational effort and hence fast methods are desirable. We develop and analyze Boolean approximation methods using the piecewise constant functions for solving integral equations of the second kind on a unit cube in R^d, including the Boolean Galerkin method and the Boolean collocation method. These schemes are based on an idea from Boolean sum approximation to obtain a linear combination of multiple coarse levels of approximations. We prove that these schemes provide fast computational methods. Specifically, they have convergence in order $\mathcal{O}(h \log^{d-1}(h^{-1}))$, with computational cost in order $\mathcal{O}(h^{-1} \log^{d-1}(h^{-1}))$, as $h \to 0$, where h is the mesh size used in the methods. For the special case when $d = 2$, we develop an iterated Boolean Galerkin method and prove the super-convergence property of this method.

1. Introduction. Integral equations of the second kind with smooth kernels in *high* dimensions have important applications in many areas such as physics, engineering and finance. Regularization of integral equations of the first kind also leads to integral equations of the second kind with smooth kernels (see, for example, [**12**] and the references cited therein). In particular, for applications of high-dimensional integral kernels in learning theory, see a recent paper [**6**]. In some areas of machine learning, a meaningful dimension is in the hundreds. Solving integral equations in high dimensions is a very challenging task

Key words and phrases. Boolean approximations, Boolean Galerkin methods, Boolean collocation methods, Boolean iterated Galerkin methods, high-dimensional Fredholm integral equations.

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