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CONVERGENCE THEOREMS AND MEASURES OF NONCOMPACTNESS FOR NONCOMPACT URYSOHN OPERATORS IN IDEAL SPACES

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ABSTRACT. A result about the uniform convergence of sequences of Urysohn operators in ideal spaces is proved when the limit operator is too singular to be compact. An estimate about the measure of noncompactness of such (weakly) singular Urysohn operators is obtained.

1. Introduction. Let S and T be σ-finite measure spaces, M a metric space, and V a Banach space. Given some function $f\colon T \times S \times M \to V$, we are interested in the corresponding Urysohn operator

$$A(f)x(t) := \int_S f(t,s,x(s))\, ds, \quad t \in T,$$

where the integral is understood in the Lebesgue-Bochner sense. If $M = V = \mathbf{R}$ and f is a so-called Carathéodory function, it is known that under some growth assumptions on f the operator $A(f)$ is compact in L_p-spaces or, more generally, in ideal spaces. These are classical results of Krasnosel'skiĭ [**4**] and Zabreĭko, see e.g., [**5, 14**]. It is also possible to weaken the growth conditions slightly [**6, 9**].

However, there are situations where f does not satisfy these growth assumptions but where one nevertheless would like to say something about the compactness of $A(f)$; if $A(f)$ is not compact, one would at least like to find good estimates for the measure of noncompactness of its image. If such a measure is sufficiently small, one can still apply, e.g., degree theory [**3**] (and in the linear case, the Fredholm alternative holds [**1**]).

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