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NONSTATIONARY WAVEFORM RELAXATION METHODS FOR ABEL INTEGRAL EQUATIONS

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ABSTRACT. In this paper the nonstationary waveform relaxation methods for Abel equations are introduced and their convergence analysis is performed. Then the fully parallel waveform relaxation methods are especially considered. Nonstationary Richardson methods are constructed in such a way to optimize the convergence rate, and a significant error estimate is proved.

1. Introduction. Large systems of Volterra integral equations with weakly singular kernels (of Abel type)

$$\begin{gathered} y(t) = f(t) + \int_0^t \frac{k(t,s,y(s))}{(t-s)^\alpha}\, ds \\ t \in [0,T], \quad 0 < \alpha < 1, \\ y, f, k \in R^d, \quad d \gg 1, \end{gathered} \tag{1.1}$$

arise in many branches of applications such as, for example, reaction-diffusion problems in small cells [**14**] as well as by the semi-discretization in space of Abel partial integral or integro-differential equations.

In order to get accurate solutions of these systems in a reasonable time frame, high performance numerical methods are required.

Methods of this kind are the waveform relaxation methods that have been recently developed by some of the authors for systems of Volterra equations both with regular kernels [**6, 8, 10–12, 13**] and with weakly singular kernel [**4**].

The waveform relaxation methods (WR methods) for the system (1.1) are introduced using a suitable function $\mathcal{G} = \mathcal{G}(t,s,u,v)$ such that

$$\mathcal{G}(t,s,u,u) = k(t,s,u). \tag{1.2}$$

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