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POSITIVE-DEFINITENESS, INTEGRAL EQUATIONS AND FOURIER TRANSFORMS

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ABSTRACT. We show that positive definite kernel functions $k(x,y)$, if continuous and integrable along the main diagonal, coincide with kernels of positive integral operators in $L^2(\mathbf{R})$. Such an operator is shown to be compact; under the further assumption $k(x,x) \to 0$ as $|x| \to \infty$ it is also trace class and the corresponding bilinear series converges absolutely and uniformly. If $k^{1/2}(x,x) \in L^1(\mathbf{R})$, all these results are carried through to a 'rotated' Fourier transform: $\hat{k}(\nu_1, -\nu_2)$ is the kernel of a compact positive operator and is represented by the absolutely and uniformly convergent series of Fourier transforms of eigenfunctions. The trace of the operator is an invariant under Fourier transforms.

1. Introduction. A number of recent applications renewed interest in the study of 'positive definite matrices' in the sense of Moore or, as we shall call them below, a positive definite kernel functions. In signal processing many physical phenomena are modeled by random processes; for second order processes, reconstruction of the signal by sampling requires consideration of the autocorrelation function both in the time and frequency domains. This function is by construction a positive definite kernel function [**3**]. In a similar vein, the theory of machine learning leads to similar questions [**4**]. It thus becomes a problem of interest for applications to study this class of functions and their Fourier transforms.

The aim of this paper is to carry out this study. We show in Section 3 that, under the assumptions of continuity and summability along the diagonal, a positive definite kernel function $k(x,y)$ is the kernel of a positive integral operator in $L^2(\mathbf{R})$. We show that positivity implies that this operator is Hilbert-Schmidt and thus necessarily compact. It then follows from standard spectral theory that k is expressed by an L^2

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