JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 16, Number 1, Spring 2004

A NOTE ON THE FREDHOLM PROPERTY OF PARTIAL INTEGRAL EQUATIONS OF ROMANOVSKIJ TYPE

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ABSTRACT. Some conditions are given, both necessary and sufficient, under which a partial integral equation of Romanovskij type defines a Fredholm operator of index zero in the space of continuous functions.

In 1932, Romanovskij [**6**] has described a problem in the theory of Markov chains with two-sided link which leads to an equation of the form

$$x(t,s) = Rx(t,s) + f(t,s), \quad (t,s) \in D := [a,b] \times [a,b], \tag{1}$$

where R is the linear operator defined by

$$Rx(t,s) = \int_a^b m(t,s,\sigma)x(\sigma,t)\,d\sigma \tag{2}$$

which contains some continuous or measurable kernel function $m : D \times [a,b] \to \mathbf{R}$. A particular feature of the operator (2) is that first the two variables in the unknown integrand x are inverted, and afterwards the integration is carried out with respect to the first variable.

Equation (1) has been studied for continuous kernel functions in [**6**] by means of Fredholm determinants. In this connection it turned out that many results on the problem (1) are quite different from classical results on Fredholm integral equations, mainly due to the fact that the operator (2) is not compact and not even an integral operator. It is natural (and now common sense) to call operators of the form (2) *partial integral operators*, inasmuch as the integration in (2) is carried out only with respect to one variable while the other variables are "frozen."

AMS *Mathematics Subject Classification.* 45A05, 45B05, 45P05, 46E15, 47A53, 47B38, 47G10.
Key words and phrases. Partial integral operator, partial integral equation, Romanovskij type equation, Fredholm operator.
Received by the editors on August 11, 2003.