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DISCONTINUOUS BOUNDARY-VALUE PROBLEMS: EXPANSION AND SAMPLING THEOREMS

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ABSTRACT. This paper is devoted to the derivation of expansion and sampling theorems associated with nth order discontinuous eigenvalue problems defined on $[-1,1]$, illustrated with detailed examples. The problem consists of nth order differential expressions and n boundary and n compatibility conditions at $x=0$. The differential expressions are defined, in general, in two different ways throughout $[-1,1]$. We derive an eigenfunction expansion theorem for the Green's function of the problem as well as a theorem of uniform convergence of the Birkhoff series of a certain class of functions. Then we derive a sampling theorem for integral transforms whose kernels are the product of the Green's function and the characteristic determinant of the problem.

1. Introduction. In [**24**] a sampling theorem associated with the discontinuous Sturm-Liouville problem

$$l^{(2)}y := -y'' + q(x)y = \lambda y, \quad 0 \le x \le \pi, \tag{1.1}$$

$$hy(0) - y'(0) = hy(\pi) + y'(\pi) = 0, \tag{1.2}$$

with two symmetric discontinuities at $d_1 = d$, $0 < d < \pi/2$ and $d_2 := \pi - d$ is studied, where the following jump conditions are satisfied

$$y(d_1^+) = ay(d_1^-), \quad y'(d_1^+) = a^{-1}y'(d_1^-) + by(d_1^-), \tag{1.3}$$

$$y(d_2^-) = ay(d_2^-), \quad y'(d_2^-) = a^{-1}y'(d_2^+) - by(d_2^+). \tag{1.4}$$

Here h, a, b are real numbers with $a > 0$ and $q(\cdot)$ is an $L^1(0,\pi)$-real valued function. The eigenfunction expansion theorem associated with

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