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SUPERCONVERGENCE IN THE MAXIMUM NORM OF A CLASS OF PIECEWISE POLYNOMIAL COLLOCATION METHODS FOR SOLVING LINEAR WEAKLY SINGULAR VOLTERRA INTEGRO-DIFFERENTIAL EQUATIONS

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ABSTRACT. A piecewise polynomial collocation method on graded grids for solving linear weakly singular integro-differential equations of Volterra type is studied. It is shown that a special choice of collocation parameters improves the convergence rate of the method, the error estimates for all values of the nonuniformity parameter of the grid are obtained.

1. Introduction and the main result. We consider the linear integro-differential equation

$$y'(t) = p(t)y(t) + q(t) + \int_0^t K(t,s)y(s)\,ds, \quad t \in [0,T], \quad T > 0, \tag{1}$$

with a given initial condition $y(0) = y_0$, $y_0 \in \mathbf{R} = (-\infty, \infty)$. We assume that

$$p, q \in C^{k,\nu}(0,T], \quad K \in \mathcal{W}^{k,\nu}(\Delta_T), \quad k \in \mathbf{N} = \{1, 2, \dots\}, \quad \nu \in \mathbf{R} \setminus \mathbf{Z},\ \nu < 1. \tag{2}$$

Here $C^{k,\nu}(0,T]$, $k \in \mathbf{N}$, $\nu < 1$, is defined as collection of all k times continuously differentiable functions $x : (0,T] \to \mathbf{R}$ such that the estimation

$$\left|x^{(j)}(t)\right| \le c_j \begin{cases} 1 & \text{if } j < 1-\nu, \\ t^{1-\nu-j} & \text{if } j > 1-\nu \end{cases}$$

holds with a constant $c_j = c_j(x)$ for all $t \in (0,T]$ and $j = 0, 1, \dots, k$. The set $\mathcal{W}^{k,\nu}(\Delta_T)$, with $k \in \mathbf{N}$, $\nu < 1$, $\Delta_T = \{(t,s) \in \mathbf{R}^2 : 0 \le t \le T,$

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