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SOLUTIONS OF HAMMERSTEIN INTEGRAL EQUATIONS VIA A VARIATIONAL PRINCIPLE

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ABSTRACT. We study solutions of the nonlinear Hammerstein integral equation with changing-sign kernels by using a variational principle of Ricceri and critical points theory techniques. Combining the effects of a sublinear and superlinear nonlinear terms we establish new existence and multiplicity results for the equation. As an application we consider a semilinear Dirichlet problem for polyharmonic elliptic operators.

1. Introduction. We study the solvability of the nonlinear Hammerstein integral equation

$$u(x) = \int_\Omega k(x,y) f(y, u(y))\, dy, \tag{1}$$

where $\Omega \subset \mathbf{R}^N$ is a bounded domain, i.e., open connected set, $k(x,y) : \Omega\times\Omega \to \mathbf{R}$ is a measurable and symmetric kernel and $f(x,u) : \Omega\times\mathbf{R} \to \mathbf{R}$ is a Carathéodory function, that is, $f(x,u)$ is measurable for each $u \in \mathbf{R}$ and continuous for almost all $x \in \Omega$.

The Hammerstein equation (1) appeared in the earlier 30s as a general model for study of semi-linear boundary-value problems. The kernel $k(x,y)$ typically arises as the Green function of a differential operator. Green functions of specific boundary value problems admit lots of specific properties like positivity, maximum principles, pointwise estimates, etc., depending on the structure of the differential expression in the data and boundary conditions. If the kernel $k(x,y)$ is positive, then methods of positive operators are applicable to study solutions of (1). The advantage of positivity methods is that in many cases they allow to obtain not only existence but also rather precise information about a location of solutions, for example, between explicitly constructed sub- and super-solutions of the equation, see, e.g., [**1, 8**].

Another classical method to study equation (1) is variational. If the kernel $k(x,y)$ is symmetric, one can associate to (1) a functional J on

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