JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 15, Number 4, Winter 2003

EXISTENCE RESULTS FOR BV-SOLUTIONS OF NONLINEAR INTEGRAL EQUATIONS

DARIUSZ BUGAJEWSKI AND DONAL O'REGAN

ABSTRACT. In this paper we deal with the existence of global bounded variation (BV) solutions as well as continuous BV-solutions of nonlinear Hammerstein and Volterra-Hammerstein integral equations formulated in terms of the Lebesgue and the Denjoy-Perron integral. The method of proof is based on an application of the Leray-Schauder alternative for contractions.

1. Introduction. Functions of bounded variation appear frequently as solutions to many integral equations which describe concrete physical phenomena. This fact motivated us to investigate bounded variation solutions as well as continuous bounded variation solutions of the Hammerstein type integral equation

$$x(t) = g(t) + \int_0^T K(t,s)f(x(s))\,ds, \quad \text{for } t \in I = [0,T], \tag{1}$$

and the Volterra-Hammerstein integral equation

$$x(t) = g(t) + \int_0^t K(t,s)f(x(s))\,ds, \quad \text{for } t \in I, \tag{2}$$

where I is a compact interval in $\mathbf{R}$. First we will investigate equations (1) and (2) with the Lebesgue integral and then later with the Denjoy-Perron integral.

The theory of the Denjoy-Perron integral gives a pure theoretical motivation for the need to investigate BV-solutions of equations (1) and (2). More precisely, it is well-known that if $h : I \to \mathbf{R}$ is any function integrable in the Denjoy-Perron sense and ϕ is a function of bounded variation, then $h\phi$ is also integrable in this sense, see [**5**] for details.

Received by the editors on December 31, 2002, and in revised form on August 10, 2003.