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pTH MEAN INTEGRABILITY AND ALMOST SURE ASYMPTOTIC STABILITY OF SOLUTIONS OF ITÔ-VOLTERRA EQUATIONS

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ABSTRACT. This paper studies the pathwise asymptotic stability and integrability of the zero solution of a finite-dimensional Itô-Volterra equation. Under Lipschitz conditions on the state-dependent functions, and with continuity and integrability required of the kernels, it is shown that any solution which is pth mean integrable for some $p \geq 2$ is pth mean-asymptotically stable, and also pth mean integrable and asymptotically stable, almost surely. If there is no delay-dependent term in the volatility, the same result can be shown for $p \geq 1$. Examples which illustrate the usefulness of these results are presented, and extensions to other classes of functional differential equations are discussed.

1. Introduction. Much research in recent years has focused on the almost sure exponential asymptotic stability of solutions of stochastic differential equations and stochastic delay differential equations with bounded delay, with several recent monographs appearing by Mao [**11, 12**]. However, less attention has been devoted to the almost sure exponential asymptotic stability of Itô-Volterra equations, where the delay is unbounded. It has been shown in Appleby and Reynolds [**3**] for nonlinear scalar time-homogeneous Itô-Volterra equations of the form

$$dX(t) = \left(f(X(t)) + \int_0^t k(t-s)g(X(s))\,ds \right) dt + h(X(t))\,dB(t) \tag{1}$$

for which the kernel k is strictly positive and satisfies a nonexponential asymptotic condition, the solution satisfies

$$\limsup_{t\to\infty} |X(t)|e^{\varepsilon t} = \infty, \quad \text{a.e. on } A$$

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