FREDHOLMNESS OF SINGULAR INTEGRAL OPERATORS WITH PIECEWISE CONTINUOUS COEFFICIENTS ON WEIGHTED BANACH FUNCTION SPACES

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ABSTRACT. We prove necessary conditions for the Fredholmness of singular integral operators with piecewise continuous coefficients on weighted Banach function spaces. These conditions are formulated in terms of indices of submultiplicative functions associated with local properties of the space, of the curve, and of the weight. As an example, we consider weighted Nakano spaces $L_w^{p(\cdot)}$ (weighted Lebesgue spaces with variable exponent). Moreover, our necessary conditions become also sufficient for weighted Nakano spaces over nice curves whenever w is a Khvedelidze weight, and the variable exponent $p(t)$ satisfies the estimate

$$|p(\tau) - p(t)| \leq A/(-\log|\tau - t|).$$

1. Introduction. Let Γ be a Jordan curve, that is, a curve that homeomorphic to a circle. We suppose that Γ is rectifiable. We equip Γ with Lebesgue length measure $|d\tau|$ and the counter-clockwise orientation. The *Cauchy singular integral* of a measurable function $f : \Gamma \to \mathbf{C}$ is defined by

$$(Sf)(t) := \lim_{R \to 0} \frac{1}{\pi i} \int_{\Gamma \setminus \Gamma(t,R)} \frac{f(\tau)}{\tau - t} d\tau, \quad t \in \Gamma,$$

where the "portion" $\Gamma(t, R)$ is

$$\Gamma(t, R) := \{\tau \in \Gamma : |\tau - t| < R\}, \quad R > 0.$$