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ASYMPTOTIC BEHAVIOR OF SOLUTIONS OF A CONSERVED PHASE-FIELD SYSTEM WITH MEMORY

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ABSTRACT. We show that any global bounded solution of a conserved phase-field model with memory terms converges to a single stationary state as time goes to infinity. The idea of analyticity plays a key role in our analysis.

1. Introduction. The time evolution of the phase variable $\chi(t,x)$ and the temperature $\vartheta(t,x)$ in the conserved phase-field model proposed by Caginalp [**7**] is governed by the system of differential equations:

$$\tau\partial_t\chi = -\xi^2\Delta(\xi^2\Delta\chi - W'(\chi) + \lambda\vartheta), \tag{1.1}$$

$$\partial_t(\vartheta + \lambda\chi) + \operatorname{div}\mathbf{q} = 0, \tag{1.2}$$

where W is typically a double-well potential, λ is a positive constant representing the latent heat, $\tau > 0$ and $\xi > 0$ stands for a relaxation time and correlation length, respectively, and $\mathbf{q}$ denotes the heat flux. Here we shall assume that $\mathbf{q}$ is determined by the linearized Coleman-Gurtin [**8**] constitutive relation:

$$\mathbf{q} = -k_I\nabla\vartheta - k * \nabla\vartheta, \tag{1.3}$$

where the constant $k_I > 0$ is the instantaneous heat conductivity, k is a suitable dissipative kernel, and the symbol $*$ denotes the time convolution:

$$k * v(t) = \int_0^\infty k(s)v(t-s)\,ds.$$

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