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STABILITY OF QUALOCATION METHODS FOR ELLIPTIC BOUNDARY INTEGRAL EQUATIONS

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ABSTRACT. I.H. Sloan and W.L. Wendland analyzed thoroughly qualocation with spline trial and test spaces in the paper *Qualocation methods for elliptic boundary integral equations* [**9**]. They derived a criterion for stability and computed numerically weights and knots for some methods which are suitable for equations in which the even symbol part of the operator dominates and other methods for a dominant odd part. Stability of these methods was ensured by numerical computations. We simplify their stability result yielding a representation which allows to prove which J-point quadrature rules are leading to stable qualocation and which are not. Furthermore, the existence of at least one stable method follows for any J.

1. Introduction In previous papers [**2**], [**9**], [**10**], e.g., the qualocation method has been introduced and applied to a large class of boundary integral equations on smooth curves in the plane. Here we study the stability of this method and of tolerant qualocation, cf. [**11**], as well. Hence, we need the same assumptions as Sloan and Wendland, i.e., we suppose that the equations are expressible in the form

$$(b_+L_+ + b_-L_- + K)u = f,$$

where $L := b_+L_+ + b_-L_- + K$ is a classical periodic pseudodifferential operator of order β. We assume that $L : H^\tau \to H^{\tau-\beta}$ defines an isomorphic mapping for any $\tau \in \mathbf{R}$, $H^\tau = H^\tau[0,1]$ being the Sobolev-space of 1-periodic functions. L_+ and L_- are the even and the odd part of L, respectively. *Both* of them may have the order β. K is a sufficiently smoothing perturbation. b_+, b_- are smooth, 1-periodic coefficients. f is a given 1-periodic function in $H^{\tau_0-\beta}$ for some $\tau_0 > \beta + \frac{1}{2}$. Moreover, we assume either that L is uniformly strongly elliptic or that L is uniformly oddly elliptic, i.e., either the even symbol part of the operator dominates or the odd symbol part, see [**10**] again for a formal definition.

We consider spline qualocation on a family of uniform meshes

$$\{x_k := k/N,\ k = 0(1)N-1\}.$$