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THE RADIOSITY EQUATION ON CERTAIN SPACES OF CONTINUOUS FUNCTIONS AND ITS NUMERICAL SOLUTION

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ABSTRACT. In this article we study the radiosity equation on a polyhedral surface S in $\mathbf{R}^3$. We construct a special space of continuous functions on S where we can prove the existence of a unique solution of the radiosity equation. These results enable us to construct grids for the numerical approximation of the solution which guarantee convergence in the maximum norm for the collocation method. In a last section we present numerical results which confirm our theoretical prediction and these results also show that graded meshes will increase the order of convergence.

1. Introduction. The exchange of energy by radiation is an important physical mechanism for heat transfer, see [**19, 22**–**24**], and for the calculation of 3D pictures in computer science, see [**7, 20, 25**]. For the heat transfer the exchange of energy by radiation is only one transport mechanism, besides diffusion and convection. The relative importance of these mechanisms depends on material properties and the surface temperature.

In contrast to this the radiation is the only process to consider in the calculation of 3D scenes in computer graphics. Here the sources of radiation are prescribed by lamps, which are distributed on the surface. In general the emitted radiation at every point depends on the direction. But in this article we will consider only surfaces where the radiance, see [**19**] for a definition, fulfills the Lambertian cosine law, which means that the radiance is constant in all directions. So this emitted radiation at every point can be characterized by a scalar, which determines the density of the emitted energy, and this quantity is called radiosity. To be consistent, we also must assume that all radiation sources are diffusive emitters. The resulting model does not contain any specular

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