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AN ASYMPTOTIC SERIES APPROACH TO QUALOCATION METHODS

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ABSTRACT. In this paper we develop an asymptotic analysis of qualocation methods when applied to a particular class of pseudodifferential equations. The main result is the existence of an error expansion between the numerical solution and an optimal projection over the splines. As by-products of this expansion we obtain some estimations of pointwise convergence and an asymptotic expansion between the exact and the numerical solution under the action of regularizing operators. In addition to this, using the error expansion we deduce sufficient conditions to obtain qualocation methods of higher order for some particular equations. Finally, we give a numerical experiment which corroborates the theoretical results.

1. Introduction. This paper is devoted to a full asymptotic analysis of qualocation methods for a certain class of periodic pseudodifferential equations. The class of equations (associated to what we will call expandable operators) we will be working on includes boundary integral equations [**3**] on smooth closed curves of the plane. In fact, it can be proved that the set of operators in our equations is that of periodic classical pseudodifferential of integer order [**11, 13, 19**].

Qualocation methods (qualocation is a compression of quadrature-modified collocation) form a recent family of numerical methods for the approximation of pseudodifferential equations. This family consists of three different groups of methods. The first group is that of semi-discretizations of spline-spline Petrov-Galerkin schemes via substitution of the test integration process by a suitable quadrature rule [**8, 28**]. The second one includes quadrature approximation of outer integrals in a spline-trigonometric Petrov-Galerkin method [**22, 26**]. Finally, the so-called "tolerant" qualocation only discretizes the test integral in the bilinear form but not in the righthand side (cf. [**30, 31**]). The paper [**25**] surveys recent advances on the analysis of this family of methods.

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