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ON THE KONTOROVICH-LEBEDEV TRANSFORMATION

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ABSTRACT. Further developments of the results on the Kontorovich-Lebedev integral transformation are given. In particular, properties of the boundedness, compactness in $L_{\nu,p}$, $1 \leq p \leq \infty$, $\nu < 1$, are established. The Bochner type representation theorem is proved. An example of the Fredholm integral equation associated with the Kontorovich-Lebedev operator is considered.

1. Introduction and auxiliary results. In this paper we investigate mapping properties of the Kontorovich-Lebedev operator [**3**], [**5**], [**11**]

$$K_{ir}[f] = \sqrt{\frac{2}{\pi}} \int_0^\infty K_{i\tau}(x) f(x)\, dx, \quad \tau \in \mathbf{R}_+, \tag{1.1}$$

which is associated with the Macdonald function $K_{i\tau}$ as the kernel [**1**] in its natural domain of definition $f \in L^0 \equiv L_1(\mathbf{R}_+; K_0(x)\, dx)$, i.e.,

$$L^0 := \Big\{ f : \int_0^\infty K_0(x)|f(x)|\, dx < \infty \Big\}. \tag{1.2}$$

In particular, it contains all spaces $L^\alpha \equiv L_1(\mathbf{R}_+; K_0(\alpha x)\, dx)$, $0 < \alpha \leq 1$ and $L_{\nu,p}(\mathbf{R}_+)$, $\nu < 1$, $1 \leq p \leq \infty$, with the norms

$$\|f\|_{L^\alpha} = \int_0^\infty K_0(\alpha x)|f(x)|\, dx < \infty, \tag{1.3}$$

$$\|f\|_{\nu,p} = \left(\int_0^\infty x^{\nu p - 1} |f(x)|^p\, dx \right)^{\frac{1}{p}} < \infty, \tag{1.4}$$
$$\|f\|_{\nu,\infty} = \operatorname{ess\,sup}_{x \geq 0} |x^\nu f(x)| < \infty.$$

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