

NON-AUTONOMOUS INTEGRODIFFERENTIAL EQUATIONS WITH NON-LOCAL CONDITIONS

JAMES H. LIU AND KHALIL EZZINBI

ABSTRACT. Recent results concerning the existence and uniqueness of mild and classical solutions for non-local Cauchy problems are extended to the following non-autonomous semi-linear integrodifferential equation

$$u'(t) = A(t) \left[u(t) + \int_0^t F(t, s)u(s) ds \right] + f(t, u(t)),$$
$$0 \leq t \leq T,$$
$$u(0) + g(t_1, \dots, t_p, u(t_1), \dots, u(t_p)) = u_0,$$

in a Banach space X , with $A(\cdot)$ the generators of strongly continuous semigroups. The non-local condition can be applied in physics with better effect than the classical Cauchy problem $u(0) = u_0$, since more measurements at t_i s are allowed. The variation of constants formula for solutions via a resolvent operator is first derived in order to carry out the study.

1. Introduction. In this paper, we will study the existence and uniqueness of mild and classical solutions for the non-autonomous semi-linear integrodifferential equation with non-local Cauchy problems.

To begin, let us consider the following semi-linear problem

$$(1.1) \quad u'(t) = Au(t) + f(t, u(t)), \quad 0 \leq t \leq T,$$

$$(1.2) \quad u(0) = u_0,$$

in a Banach space X , with A the generator of a strongly continuous semigroup $T(\cdot)$. Here in (1.2), $u(0) = u_0$ is referred to as the initial value problem, or Cauchy problem.

AMS *Mathematics Subject Classification.* 45K, 34G.

Key words and phrases. Non-local conditions, non-autonomous integrodifferential equations, resolvent operators.

Copyright ©2003 Rocky Mountain Mathematics Consortium