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ON THE LIMIT MEASURE TO STOCHASTIC VOLTERRA EQUATIONS

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ABSTRACT. The paper is concerned with a limit measure of stochastic Volterra equation driven by a spatially homogeneous Wiener process with values in the space of real tempered distributions. Necessary and sufficient conditions for the existence of the limit measure are provided and a form of any limit measure is given as well.

1. Introduction. In the paper we investigate the limit measure to the stochastic Volterra equation of the form

$$X(t,\theta) = \int_0^t v(t-\tau)AX(\tau,\theta)\,d\tau + X_0(\theta) + W(t,\theta), \tag{1}$$

where $t \in \mathbf{R}_+$, $\theta \in \mathbf{R}^d$, $v \in L^1_{\mathrm{loc}}(\mathbf{R}_+)$, $X_0 \in S'(\mathbf{R}^d)$ and W is a spatially homogeneous Wiener process which takes values in the space of real, tempered distributions $S'(\mathbf{R}^d)$. The class of operators A contains, in particular, the Laplace operator Δ and its fractional powers $-(-\Delta)^{\alpha/2}$, $\alpha \in (0,2]$.

Description of asymptotic properties of solutions to stochastic evolution equations in finite dimensional spaces and Hilbert spaces is well known and has been collected in the monograph [**7**]. Recently this problem has been studied for generalized Langevin equations in conuclear spaces by Bojdecki and Jakubowski [**1**]. The question of existence of invariant and limit measures in the space of distributions seems to be particularly interesting, especially for stochastic Volterra equations, because this class of equations is not well investigated.

In the paper we give a necessary and sufficient condition for the existence of a limit measure and describe all limit measures to the equation (1). Our results are in a sense analogous to those formulated for the

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