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THREE-POINT BOUNDARY VALUE PROBLEMS WITH SOLUTIONS THAT CHANGE SIGN

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ABSTRACT. Using the theory of fixed point index, we give new results for some three-point boundary value problems. In particular we study problems where the associated integral equation has a kernel that changes sign, so that positive solutions cannot exist. We obtain the existence of at least one or of multiple nonzero solutions.

1. Introduction. In this paper we study the existence of nonzero solutions of second order differential equations of the form

$$u''(t) + g(t)f(u(t)) = 0, \quad 0 < t < 1 \tag{1.1}$$

under one of the boundary conditions (BC's)

$$u'(0) = 0,\ \alpha u(\eta) = u(1), \quad 0 < \eta < 1, \tag{1.2a}$$

$$u(0) = 0,\ \alpha u(\eta) = u(1), \quad 0 < \eta < 1. \tag{1.2b}$$

These so-called three-point boundary value problems (BVP's), and more general m-point BVP's, are examples of nonlocal boundary conditions whose study has been motivated by work of Bitsadze and Samarskii and Il'in and Moiseev [**6**]. In recent years, the existence of solutions of equations more general than (1.1) has been thoroughly studied by Gupta, et al., see, for example, [**3, 4**] and the references therein.

Other authors, for example Ma [**9**] and Webb [**11**], have studied the existence of one or of multiple positive solutions when $0 < \alpha < 1$ for (1.2a) and $0 < \alpha\eta < 1$ for (1.2b).

One approach is to write the BVP as an equivalent Hammerstein integral equation

$$u(t) = \int_0^1 k(t,s)g(s)f(u(s))\,ds := Tu(t) \tag{1.3}$$

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