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SOLVABILITY AND SPECTRAL PROPERTIES OF INTEGRAL EQUATIONS ON THE REAL LINE: II. L^p-SPACES AND APPLICATIONS

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Dedicated to Professor Ian Sloan on the occasion of his 65th birthday.

ABSTRACT. We consider the solvability of linear integral equations on the real line, in operator form $(\lambda - K)\phi = \psi$, where $\lambda \in \mathbf{C}$ and K is an integral operator. We impose conditions on the kernel, k, of K which ensure that K is bounded as an operator on $L^p(\mathbf{R})$, $1 \leq p \leq \infty$, and on $BC(\mathbf{R})$. We establish conditions on families of operators, $\{K_k : k \in W\}$, which ensure that if $\lambda \neq 0$ and $\lambda\phi = K_k\phi$ has only the trivial solution in $BC(\mathbf{R})$, for all $k \in W$, then for $1 \leq p \leq \infty$, $(\lambda - K)\phi = \psi$ has exactly one solution $\phi \in L^p(\mathbf{R})$ for every $k \in W$ and $\psi \in L^p(\mathbf{R})$. The results of considerable generality apply in particular to kernels of the form $k(s,t) = \kappa(s-t)z(t)$ and $k(s,t) = \tilde{\kappa}(s-t)\tilde{z}(s,t)$, where $\kappa, \tilde{\kappa} \in L^1(\mathbf{R})$, $z \in L^\infty(\mathbf{R})$, $\tilde{z} \in BC(\mathbf{R}^2)$ and $\tilde{\kappa}(s) = O(s^{-b})$ as $|s| \to \infty$, for some $b > 1$. As a significant application we consider the problem of acoustic scattering by a sound-soft, unbounded one-dimensional rough surface which we reformulate as a second kind boundary integral equation. Combining the general results of earlier sections with a uniqueness result for the boundary value problem, we establish that the integral equation is well-posed as an equation on $L^p(\mathbf{R})$, $1 \leq p \leq \infty$, and on weighted spaces of continuous functions.

1. Introduction. We consider in this paper integral equations of the form

$$\lambda\phi(s) - \int_{-\infty}^{+\infty} k(s,t)\phi(t)\,dt = \psi(s), \quad s \in \mathbf{R}, \tag{1.1}$$

where $\lambda \in \mathbf{C}$, the functions $k : \mathbf{R}^2 \to \mathbf{C}$ and ψ are assumed known and ϕ is the solution to be determined. Define the integral operator K by

$$K\psi(s) = \int_{-\infty}^{+\infty} k(s,t)\psi(t)\,dt, \quad s \in \mathbf{R}. \tag{1.2}$$

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